



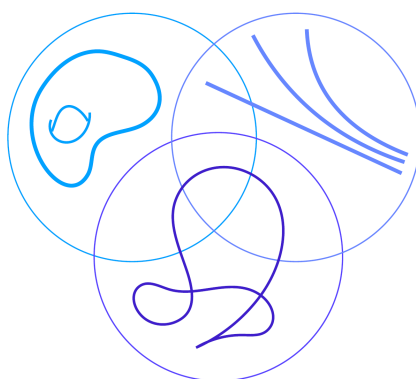
# Annual Report 2020

(1 April 2020 - 31 December 2020, "Year 1")

Copenhagen Centre for Geometry and Topology  
DNRF151

Department of Mathematical Sciences  
University of Copenhagen

Established April 1, 2020



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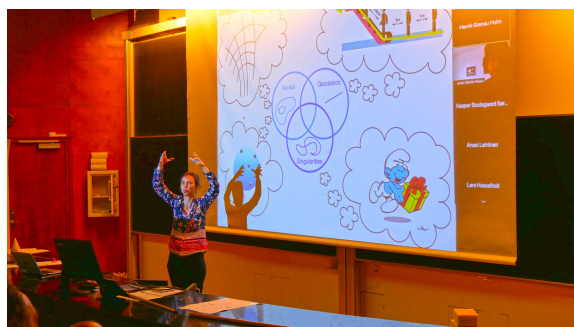
# GEOTOP: CENTRE HIGHLIGHTS 2020

The *Copenhagen Centre for Geometry and Topology* opened April 1, 2020, as the world realised it was facing a pandemic. The centre quickly adapted to these new circumstances and learned to swiftly move between on-site, online, or hybrid ways of being. By the end of the year, the centre counted 10 permanent members, 3 associated members, 13 postdocs, and 15 PhD students.



## RESEARCH

We made significant progress on various aspects of our research program. Highlights include: Land and 8 coauthors finished a substantial 3-volume manuscript on Hermitian  $K$ -theory, simultaneously laying new and improved foundations and settling old questions. In particular, they gave an essentially complete calculation of symplectic and orthogonal  $K$ -theory of  $\mathbb{Z}$ , the ring of integers. Also, Randal-Williams and Kupers studied moduli spaces of high-dimensional disks. They made substantial progress in understanding their rational homotopy theory and introduced a new method that gives hope of eventually understanding the entire rational homotopy type of these fundamental building blocks.



## ACTIVITIES

After a successful Opening Symposium, the year was marked by a highly popular masterclass on *Condensed Mathematics* by 2018 Fields medalist Scholze and centre member Clausen. The speakers laid the foundations of this new subject. The recordings of the event can be seen on our YouTube channel (387 subscribers). This will serve as a long-term reference for those who want to learn the subject. The first lecture has already had 7,945 views!

## RECOGNITION

Grodal and Wahl were both elected members of the Royal Danish Academy of Science and Letters, and Adiprasito received the prestigious EMS prize of the European Mathematical Society.



Please visit [geotop.math.ku.dk](http://geotop.math.ku.dk) for more information.

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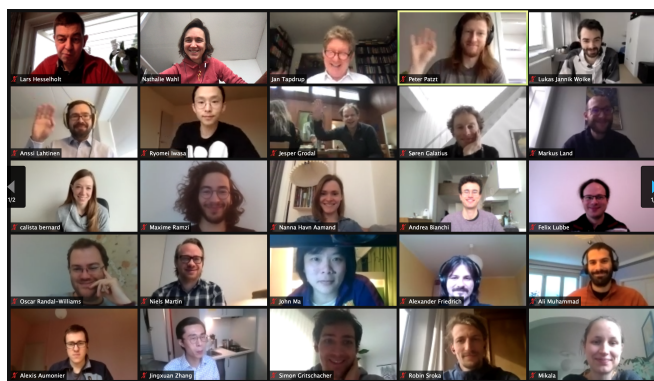
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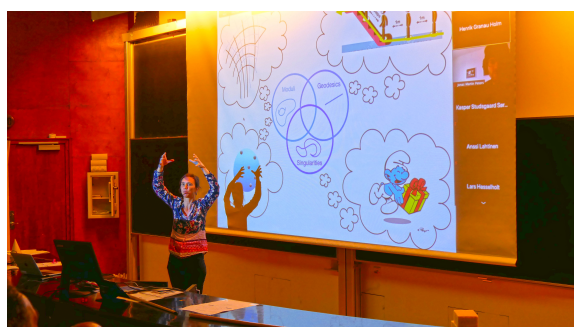
# GEOTOP CENTERHØJDEPUNKTER 2020

Centret *Copenhagen Centre for Geometry and Topology* startede 1. april 2020, netop som verden vågnede op til pandemiens realiteter. Det nye center tilpassede sig hurtigt de nye omstændigheder og lærte at veksle mellem og kombinere fysisk og virtuel tilstedeværelse. Ved årets udgang talte centeret 10 permanente medlemmer, 3 associerede medlemmer, 13 postdocs samt 15 ph.d.-studerende.



## FORSKNING

Centerets medarbejdere gjorde flere betydelige fremskridt i relation til forskningsprogrammet i løbet af det første år. Et af højdepunkterne var et langt manuskript om Hermitisk  $K$ -teori af Land og 8 medforfattere. I dette udvikles et nyt og forbedret fundament med bredere anvendelsesmuligheder, men det afklarer også gamle formodninger. Ikke mindst gives en stort set fuldstændig bestemmelse af symplektisk og ortogonal  $K$ -teori af de hele tal  $\mathbb{Z}$ . Et andet højdepunkt er Kupers og Randal-Williams studie af modulirum af højt-dimensionale diske. Der introducerede de en ny metode som allerede har givet markante fremskridt i studiet af disse fundamentale objekter. Dette giver håb for at vi med tiden vil opnå en fuldstændig forståelse af deres rationale homotopitype.



## AKTIVITETER

Centeret afholdt et vellykket åbnings Symposium, og senere på året en masterclass om *Condensed Mathematics* som tiltrak meget opmærksomhed. Forelæserne var Scholze, som modtog Fields-medaljen som 30-årig i 2018, og centermedlem Clausen. Masterclassen blev optaget og er lagt på centerets YouTube-kanal (387 abonnenter), hvor den vil tjene som kilde for enhver som ønsker at lære dette nye område. Første forelæsning er vist 7.945 gange!

## PRISER OG ANERKENDELSER

Grodal og Wahl blev indvalgt som medlemmer i det Kongelige Danske Videnskabsnævn, og Adiprasito modtog den prestigefyldte *EMS Prize*.



Se endvidere centerets hjemmeside [geotop.math.ku.dk](http://geotop.math.ku.dk).

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# 1 Organisation

The centre has the overall structure as outlined in the original application. **Wahl** takes the strategic decisions in coordination with **Galatius** and **N Møller**, and when relevant also with **Randal-Williams** and **Colding**. The newly hired centre administrator **Tapdrup** takes care of the centre's day-to-day running and helps out with the administrative and practical matters. This leadership team has established good collaborative routines. As anticipated, the associated members **Adiprasito**, **Clausen**, **Grodal**, **Hesselholt** and **J Møller** have benefitted the centre, particularly by engaging with and in some cases supervising PhD students and postdocs at the centre.

Our originally envisioned organisation had to adapt to continuously changing Covid-19 restrictions. Many activities moved to fully online or hybrid presence; this included the bimonthly GeoTop meetings, as well as workshops and seminars. By the beginning of the fall term, the Faculty of Science installed high-quality equipment in many lecture rooms, and the centre has purchased mobile equipment to supplement this.

## 1.1 Scientific staff

The scientific staff at the end of 2020 comprised 5 core faculty, 5 supporting faculty, 3 associated members, 13 postdocs, and 15 PhD students. Note that we are well above the contractual 6 postdocs and 6 PhD students because of substantial external funding. **N Møller** became associate professor and **Randal-Williams** was promoted to full professor at the University of Cambridge. **Randal-Williams** had to cancel his visit to the centre due to Covid-19 travel restrictions but took part in online events. On the other hand, **Colding** spent more time than initially planned in Copenhagen, in total 6 months.

The centre hired 7 new postdocs (**Avvakumov**, **Bianchi**, **Ma**, **Patz**, **Schnider**, **Woike**, **Zheng**) and included 11 postdocs who were already in Copenhagen in the centre (**Friedrich**, **Iwasa**, **Gritschacher**, **Lahtinen**, **Land**, **Li**, **Lubbe**, **Nariman**, **Pstragowski**, **Villarreal**, and **Wasserman**). A total of 5 postdocs left in 2020 (**Li** now has a postdoctoral assistant professorship at the University of Michigan, **Nariman** is a tenure-track assistant professor at Purdue, **Pstragowski** is now Benjamin Pierce fellow at Harvard, **Villarreal** is a postdoc at the University National Autonoma de Mexico, and **Wasserman** is pursuing a non-academic career).

We hired 4 new PhD students (**Cordova**, **Muhammad**, **Subramanian**, **Zhang**), and welcomed 12 students who had already started their PhD study to the centre (**Aumonier**, **Bauer**, **Borhan Tan**, **Duan**, **Jansen**, **Hunt**, **Leip**, **van der Meer**, **Poliakova**, **Steinmeyer**, **Sroka** and **Aamand**), as well as **Bernard**, who is a visiting PhD student from Stanford University. Two PhD Fellows defended their PhD Theses (**Duan** and **Hunt**).

## 1.2 Visitors

Due to the Covid-19 travel restrictions, we had only very few visitors this year: Benjamin **Brück** (Bielefeld/ETH) visited the centre for 6 months, in particular to work with **Patz**, **Sroka** and **Wahl** on homology of certain groups. A still ongoing joint project of **Brück–Patz–Sroka** resulted from this visit. Fabian **Hebestreit** (Bonn) visited the centre for two months, especially to work with **Galatius** and **Land**. **Hebestreit** and **Land** finished their long joint project on Hermitian  $K$ -theory during this visit, see Section 3. We also hosted three visiting master students: Konrad Bals, Branko Juran, and Maxime Ramzi. Ramzi will stay in Copenhagen to do a PhD, starting in September 21.

## 1.3 Administration

The establishment of the centre in 2020 necessitated new administrative practices and staff. **Tapdrup** started as centre administrator. He faced the same challenges as other new employees: starting up a new job online. But the Centre fortunately works closely together with the administration at the Department for Mathematical Sciences at the University of Copenhagen, making this situation easier. Natasha Roschier Rørdam Gulddal, an experienced accounting officer at the department, took care of budgeting, budget follow-up, and financial accounting.

## 1.4 Recruitment and gender strategy

The centre in the fall of 2020 had a call for PhD and postdoc applications together with the Department of Mathematics. We took advantage of the international platform Mathjobs for increased visibility. The postdoc call in Mathematics was a huge success with more than 500 applicants, including more than 100 applicants in topology and close to 50 in geometry. **N Møller** and **Hesselholt** played a coordinating role for the postdoc call for the whole department. For PhD fellowships we run calls for applications twice a year. The centre had around 40 PhD applicants this fall.

Diversity is on our mind when we hire. This applies to both gender diversity and more general diversity in the sense of not always going for the same type of “usual profiles”. **Wahl** is part of the department’s diversity committee. She led a 2020 taskforce to review the recruitment procedures for postdocs and PhD students with focus on diversity. This resulted in a document with guidelines and recommendations that was circulated in the department. A workshop on this topic for the Department Day was planned, but postponed due to Covid-19. We are a highly international centre, with close to 20 nationalities represented. Nevertheless, we are aware that we still have to work on our gender diversity.

## 1.5 Research integrity

The three central principles of the Danish research integrity code of conduct are *honesty*, *transparency* and *accountability*. We are fully committed to the three principles and actively work to implement them in several ways.

Mathematics has a tradition of being relatively open in the research process, with researchers sharing their results long before they are in final published form. At GeoTop we ask all centre members to put their preprints on the international preprint archive arXiv.org before submitting them to journals for publication; this promotes transparency. Likewise, staff members present preprint posted to the arXiv at bimonthly centre meetings.

We encourage everyone to publish in high standards international journals that have thorough refereeing processes. Potential issues in mathematical publishing are mistakes (these do happen and need fixing when the author becomes aware of them), failing to give proper credit to people one has received ideas from, or worst, actual plagiarism. ArXiv automatically checks for text overlap with every article submitted to the platform and adds a comment to the paper’s page in case of significant overlap.

The University of Copenhagen also runs compulsory classes on “Responsible conduct in research” for PhD students and PhD supervisors.

## 2 Research plan

We report here on the progress made on our research plan, under each of the three headings *Moduli*, *Geodesics* and *Singularities*.

### (A) Moduli

**Core:** Galatius, Randal-Williams, Wahl

**Visiting:** Hebestreit (Bonn)

**Postdocs:** Bianchi, Gritschacher, Iwasa, Land, Li, Nariman, Patzt, Pstragowski, Villarreal

**PhDs:** Aumonier, Bauer, Bernard, Borhan Tan, Cordova, Duan, Hunt, Jansen, Liep, Sroka, Subramanian, van der Meer

The focus on this part of the research has been on Goal A.1, the identification of the homotopy type of moduli spaces.

**Land** with long-term visitor **Hebestreit** and 7 other authors finished a 366-page manuscript laying new foundations of Hermitian  $K$ -theory, allowing for new connections to manifold theory, a major object of study in this part of our research program. The 9 authors also resolve older open questions, such as the Karoubi conjecture relating Hermitian  $K$ -theory to algebraic  $K$ -theory and  $L$ -theory. In a companion

manuscript, **Land**, **Hebestreit** and Nikolaus determine the homotopy types of several  $L$ -theory spectra relevant to this work. These papers are the culmination of a collaboration that goes several years back, originating in a quest to understand moduli spaces of manifolds. On a related topic, Feng, **Galatius**, and Venkatesh determined a natural action of the automorphism group of the complex numbers on a special case of Hermitian  $K$ -theory. **Galatius**, Kupers, and **Randal-Williams** submitted a paper about a new approach to Rognes' stable rank filtration.

Another advance on moduli spaces of manifolds, by **Randal-Williams** and former KU postdoc Kupers, is the computation of the rational homotopy groups of moduli spaces of the fundamental building blocks of manifolds, namely discs, in a range of degrees much larger than previously known. Their methods are based on earlier work of Kupers, and of **Galatius–Randal-Williams**, as well as representation theory. **Randal-Williams** and Bustamante studied a similar question for moduli spaces of another type of basic manifolds, namely the solid tori.

Of other developments, **Bianchi** gave an upper bound on the topological complexity of certain varieties, determining in particular the topological complexity of unordered configuration spaces of the plane. **Gritschacher** with Adem and Gómez studied the moduli space of representations of  $\mathbb{Z}^n$  in a simply connected simple Lie group  $G$ , in particular calculating its second homotopy group. **Patz** proved stability results for certain types of algebras related to symmetric groups and braids.

**Hunt** defended his thesis on decompositions and obstructions for the stable module  $\infty$ -category and **Duan**, on equivariant Euler characteristics and spaces of trees. **Jansen** handed in her PhD thesis about the reductive Borel–Serre compactification, and uploaded a preprint based on parts of her thesis.

## (B) Geodesics

**Core:** **Wahl**

**Visiting:** Brück (Bielefeld/ETH)

**Postdocs:** Avvakumov, Bianchi, Lahtinen, Schnider, Wasserman, Woike, Zheng

**PhDs:** Aamand, Poliakova, Steinmeyer

The focus on this part of the research program has been on Goal B.1, concerning geometric string topology.

String topology operations in genus zero are often modelled using the *cactus operad*. **Wahl** and co-authors introduced a new topological operad of bracketed trees, and used it to show that normalised cacti, a small model for the cactus operad, define an  $\infty$ -operad. The cactus operad is homotopy equivalent to the framed little disc operad, an operad that occurs in many places. **Woike**, together with Müller, characterised cyclic algebras over the associative and the framed little disks operad in any symmetric monoidal bicategory. In the case of linear categories subject to finiteness conditions, they show that cyclic associative and cyclic framed little disks algebras, respectively, are equivalent to pivotal Grothendieck–Verdier categories and balanced braided Grothendieck–Verdier categories. One consequence is a consistent system of handlebody group representations associated to any balanced braided Grothendieck–Verdier category inside a certain symmetric monoidal bicategory of linear categories, that generalises the handlebody part of Lyubashenko's mapping class group representations.

A very interesting development also took place in 2020: upcoming postdoc Naef by studying lens spaces has demonstrated that the coproduct in string topology is not homotopy invariant. Inspired by this, **Wahl** has defined an obstruction to homotopy invariance of a surgery flavour that may lead to unforeseen discoveries in 2021. The expertise of Professor Emeritus **Madsen** and the research team “Moduli” is expected to be particularly relevant.

## (C) Singularities

**Core:** Colding, N Møller

**Postdocs:** Friedrich, Lubbe, Ma

**PhDs:** Muhammad, Zhang

The focus on this part of the research has been on Goal C.1, that is classifying singularities in flows, and most particularly for the mean curvature flow.

Concepts and techniques developed by Colding-Minicozzi (entropy, linear stability) and Møller et al. (cone Hölder spaces, nonlinear Schauder estimates for self-shrinking singularity models) were in 2020 central as tools in important work on singularities by several other groups abroad, in their approaches to understanding singularities for the mean curvature flow, for obtaining weak versions of smooth Schönflies theorems. This line of study might lead to major breakthroughs in the understanding of singularities if a number of still outstanding technical hurdles can be handled. At Copenhagen, Møller initiated an advanced research seminar “GeoTopics” on these topics, where the state-of-the-art is explored in great depth with the postdocs and PhDs. These developments are expected to spark further progress in the field in the years to come.

In 2019, in a series of papers with his former PhD student F. Chini, Møller posed questions concerning certain “missing cases”, on the existence of mean curvature flow singularity models of ancient type which sweep out half-spaces. Such objects were then found in March 2020, just a month before GeoTop opened, by Ma, now a postdoc at GeoTop. These constructions were successful in special cases, so after arrival in Copenhagen, Ma has continued to investigate such singularities, seeking in particular to get also compact embedded examples. Møller et al. have been working on ruling out certain related types of examples. Ma likewise worked on the natural classification question, but for compact *embedded* ancient solutions in a half-space, where an expectation is that there should exist no others than the so-called Angenent’s Ovals. Furthermore, Ma uses related techniques to investigate compactness properties of Hamiltonian stationary Lagrangian surfaces in general symplectic 4-manifolds.

Lubbe has been working on higher codimension mean curvature flow, and flows of strictly spacelike hypersurfaces in cosmological spacetimes, including in the case of Lorentzian manifolds, investigating long-time existence and convergence properties. Friedrich has been working on the existence question for generalised Willmore surfaces, on a project that might have ramifications in both theoretical physics and mathematical biology.

## 3 Comments to the appendix

**Appendix—B: Conferences.** Despite the challenges caused by the pandemic, we held a successful Opening Symposium for the centre in August 2020, with in-person attendance of the University Rector, the Dean of the Faculty of Science, the CEO of DNRF, many department members, as well as online attendance from abroad. We also held a couple of small workshops over the summer, again mostly with local participants: a workshop on simplicial complexes, and a Geometry Day, both held in August at a time when the University had reopened for research activities. In November, we held a large masterclass on *Condensed Mathematics* fully online, because we expected many international participants. Indeed, about 200 persons attended the masterclass online. The videos from the masterclass since posted to YouTube have shown to be very popular, with currently 7,945 views for the first lecture on our YouTube channel! The channel already has 387 subscribers which is impressive in this specialised field. Some of the planned activities, such as the Scissors Congruence and the Mean Curvature Flow masterclasses, and the First Young Geometers Meeting, were postponed. We have faced unexpected challenges this year. But we have also learned to use digital media in a new way which is bound to change the way we do things in the future. Conferences in the future are likely to have a digital component, although nothing can replace the creative drive caused by meeting people in person.

**Appendix—C: Educational activities.** The table lists 13 graduate courses and 4 PhD courses, including a new topics course, “GeoTopics”. We postponed our plans to start a summer research experience for undergraduates due to the uncertainty of what was possible.

**Appendix—D: External funding.** The DNRF grant is, on top of the support from the Department of Mathematical Sciences, strengthened by a grant from the Independent Research Fund Denmark, two European Research Council Consolidator grants, one European Research Council Starting grant, and three Marie Skłodowska-Curie stipends. The supplementary external funding in 2020 amounted to 11.6 M DKK.

**Appendix—E: Awards.** **Grodal** and **Wahl** were elected members of the Royal Danish Academy of Sciences and Letters, while **Adiprasito** received the prestigious EMS Prize of the European Mathematical Society.

**Appendix—F: Public outreach.** Many outreach activities such as the Culture Night, Bloom Festival and the high school teacher workshop “Meet MATH” were cancelled this year. The centre had general public outreach through a newspaper article and a podcast. **N Møller** is currently collaborating with “Videnskabsklubben” to create a new large-scale national outreach program in geometry and topology for school children, and involving high school students as mentors. The planned opening is October 2021. **Galatius** is long-term member of the organising committee of the high school mathematics competition “Georg Mohr”. For undergraduates, **Grodal** gave a “Breakfast lecture” entitled “Lie, not lie!”. On the diversity side, **Bernard** started a well-received women’s lunch.

As a general outreach within the mathematical community, the centre posted on the GeoTop YouTube channel two short introductory lectures from GeoTop meetings, one on the mean curvature flow by **Ma** and one on the classification of 4-manifolds by **Land**. The videos have already received 150 views each.


**Appendix—H: Publications.** We had 30 new preprints added to the preprint server arXiv in the period April–December 2020. Publication in journals in Mathematics is a slow process. Therefore, none of our preprints have appeared in journals yet. But 6 out of the 30 preprints are already accepted for publications. An additional 4 older preprints that were revised while at least one author was at the centre are now accepted for publication. These are the 10 published articles we list in the appendix. The publication list contains the two PhD theses written by PhD students attached to the centre.

## 4 Signature

*I hereby confirm the correctness of the information concerning annual accounts, including itemisations. Also, I confirm that the compiled annual reporting, including the appendices, is correct, i.e. it is free of material misstatement or omissions, and that the administration of the funds has been secure and sound, and in accordance with the conditions of the centre agreement.*

Date  
31/3 2021

Nathalie Wahl



Professor, centre Director