



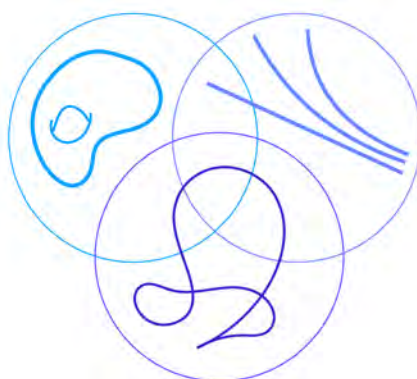
Annual Report 2021

(1 January 2021 - 31 December 2021, "Year 2")

Copenhagen Centre for Geometry and Topology
DNRF151

Department of Mathematical Sciences
University of Copenhagen

Established April 1, 2020



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GEOTOP: CENTRE HIGHLIGHTS 2021

The year 2021 was the first full year for the *Copenhagen Centre for Geometry and Topology*. The first half of the year was constrained to be mostly online, but we were ready to jump back in live as soon as the doors of the University reopened, with 3 masterclasses and workshop in hybrid mode during the second half of the year, in addition to a GeoTop IRL summer meeting at the Royal Society in June, which ended with a swim in the harbour! By the end of the year, the centre counted 11 permanent members, 3 associated members, 14 postdocs, and 12 PhD students.



RESEARCH

Highlights in the research progress include new angles of research on geodesics, that appeared both combinatorially in Avvakumov’s work, and in the study of rotationally symmetric singularities of the mean curvature flow in the work of Ma, Muhammad and NM Møller. Naef proved the surprising result that the string coproduct is not homotopy invariant, and gave indications of relations to Reidemeister torsion. The “working workshop” on configuration spaces, illustrated to the left, resulted in a list of hands-on open problems, and a few answers! 38 new preprints were posted in 2021.

ACTIVITIES

In addition to the 5 masterclasses and workshops, and the regular seminars, a new working seminar was initiated by Carmeli, solving reachable research questions over lunch on Fridays. On the outreach side, the Videnskabsklubben program *Geometry & Algorithms* had its first run, with 165 primary school students making their way through a mathematical game planned by NM Møller together with AU computer scientist KG Larsen and Copenhagen Game Lab. GeoTop also made its first appearance at the Culture Night with mathematical puzzles, soap bubbles and talks for a wide audience.



RECOGNITION

Colding was invited to be a plenary speaker of the ICM 2022, and Randal-Williams and Wahl as sectional speakers for the Topology section.

GEOTOP: CENTERHØJDEPUNKTER 2021

Året 2021 var det første hele år for *Copenhagen Centre for Geometry and Topology*. I den første halvdel af året var det meste begrænset til at foregå online, men vi stod på spring til fysisk tilstedeværelse så snart universitetets døre igen åbnedes, med 3 masterclasses og workshops på hybrid-form i det andet halvår, udover et fysisk GeoTop-sommermøde i Videnskabernes Selskab i juni, som sluttede med en svømmetur i havnen! Ved årets afslutning talte centret 11 fastansatte, 3 associerede medlemmer, 14 post-docs og 12 ph.d.-studerende.



FORSKNING

Højdepunkter i forskningen inkluderer nye tilgange til geodætiske kurver, som optrådte både kombinatorisk i Avvakumovs arbejde, og i studiet af rotationssymmetriske singulariteter i middelkrumningsflow i arbejdet af Ma, Muhammad og N.M. Møller. Naef beviste det overraskende resultat at koproduktet i string topology ikke er invariant under homotopiækvivalens og indikerede endda en relation til Reidemeister-torsion. Den “arbejdende workshop” om konfigurationsrum, illustreret her til venstre, mundede ud i en liste af håndgribelige åbne problemer samt et par svar! 38 nye preprints blev offentliggjort i løbet af 2021.

AKTIVITETER

Ud over de 5 masterclasses og workshops, samt de regelmæssige seminarer, blev et nyt arbejdsseminar startet af Carmeli, med løsning af tilgængelige forskningsproblemer over frokost på fredage. På formidlingssiden kørte programmet *Geometri & Algoritmer* i Videnskabssklubben for første gang, så 165 skoleelever tog turen igennem et matematisk spil tilrettelagt af N.M. Møller sammen med AU-datalog K.G. Larsen og Copenhagen Game Lab. GeoTop havde også sin første optræden til KulturNat med matematiske gåder, sæbebobler og foredrag.



PRISER OG ANERKENDELSER

Colding blev inviteret som plenarforedragsholder til ICM 2022, og Randal-Williams og Wahl som talere i Topologi sektionen.

1 Organisation

The centre ran throughout 2021 organised as planned, with **Wahl** taking the strategic decisions in coordination with **Galatius** and **NM Møller**, and when relevant also with **Randal-Williams** and **Colding**. They were aided on all administrative matters and day-to-day running of the centre by the centre administrator **Tapdrup**. The associated members **Adiprasito**, **Clausen**, **Grodal**, **Hesselholt** and **J Møller** have actively aided the supervision and training of the PhD students and postdocs at the centre. Despite the many travel restrictions still in place through most of 2021, **Randal-Williams** visited the centre in December and **Colding** in July, August and December.

We started the year online, patiently waiting for the university to reopen. The bimonthly GeoTop meetings, fully online for almost the first half of the year and then in hybrid mode, served as a way to keep up with everyone, and keep the information flowing through the centre.

1.1 Scientific staff

The scientific staff at the end of 2021 comprised 5 core faculty, 6 supporting faculty, 3 associated members, 14 postdocs, and 12 PhD students, again well above the contractual 6 postdocs and 6 PhD students because of substantial external funding.

The centre hired 5 new postdocs (**Biswas**, **Carmeli**, **Das**, **Jansen**, **Razlighi**) and hosted 2 new Marie Curie fellows (postdoc **Naef** and Assoc. Prof. **Kock**). A total of 5 postdocs left in 2021. (**Friedrich** is pursuing a non-academic career in the Berlin area, **Gritschacher** is now a postdoc at Ludwig-Maximilians-University Munich, **Lahtinen** is now a software developer for 3Shape in Copenhagen, **Lubbe** is now a deep learning engineer for the geometric tech start-up Grazper in Copenhagen, and **Patz** is now tenure-track assistant professor at the University of Oklahoma.)

We hired 3 new PhD students (**Asgeirsson**, **Elis**, **Ramzi**). Four PhD Fellows associated with the centre defended their PhD theses: **Bernard** (at Stanford), **Jansen**, **Poliakova** and **Sroka**. (**Bernard** is now a postdoc at the University of Minnesota, USA, **Jansen** stayed as a postdoc at the centre, **Poliakova** is now a postdoc at the University of Southern Denmark, and **Sroka** is now a postdoc at MacMaster University in Canada.)

1.2 Visitors

The centre hosted two longterm visitors in 2021, H el ene **Esnault** (Berlin) and Arthur-C esar **Le Bras** (Paris XIII), as well as about 20 non-conference related shorter term visitors, and close to 50 conference participants.

Esnault visited the centre for two months and gave a series of lectures on her breakthrough work with Groechenig on complex local systems, a major tool in the study of moduli spaces. Her visit was funded jointly with the Department of Mathematical Sciences. The lectures attracted a wide audience, ranging from PhD students to permanent staff.

Le Bras visited us for a month and a half. He gave a series of high quality and also well-attended lectures at the interface between geometry and number theory, on the subjects of the Fargues-Fontaine curve, geometrization of the local Langlands correspondence, and Fourier transform of Banach-Colmez spaces. These modern topics have completely revolutionized the local Langlands program by casting it in the framework of geometric Langlands and **Le Bras** is one of the few experts in this area. **Le Bras** started a collaboration with centre member **Clausen** on the geometrization of the theta correspondence, a cornerstone of Langlands functoriality.

1.3 Administration

Continuing from last year, **Tapdrup** was centre administrator and Natasha Roschier R ordam Gulddal took care of budgeting, budget follow-up, and financial accounting. The day-to-day administration was marked by the oscillation from online to in person attendance at the centre.

1.4 GeoTop Activities

The centre runs a Geometry and a Topology seminar, normally meeting on a weekly basis in term time, and a GeoTop meeting every other week over lunch. Standard points on the agenda of the GeoTop meetings are short research presentations, and presentations of upcoming courses and events. Other topics are also covered by the meetings. For example, a couple of GeoTop meetings this year were focused on the job market, both inside and outside of academia, and a series of 4 meetings was devoted to discussing presentation styles and techniques, with speakers being given a pre-chosen style for their presentations, such as “statement only”, “only pictures”, “only examples”, culminating at our Christmas meeting with the “theatrical” style, which **Elis** and **Carmeli** made great use of, for the joy of the whole centre!

Additional seminars and reading groups are organised by the postdocs and PhD students. A highly popular one that started in 2021 is **Carmeli**’s Homotopy theory question seminar, where participants each Friday are invited to propose questions, one of which is chosen to be discussed on for the rest of the meeting. Inspired by **Carmeli**’s seminar, **Bianchi**, **Das**, **Naef** and **Wahl** organised a “working workshop” in November on *Homotopy and Homology of configuration spaces*, where each afternoon consisted in the presentation of problems related to the theme of the workshop, working on the problems, and then meeting at the end of the afternoon to share any progress made. A list of unsolved problems was compiled at the end of the week. It was a great success!

We held our first *Young Geometers Meeting* online in April. From June onwards we were also able to host workshops and masterclasses at the University, which we ran in hybrid format, to make it possible for both speakers and participants to still take part in the event, even if they could not travel. The masterclass on *Topological field theories and factorization homology* in August was our first event with all speakers on site. It was very well received, and in fact the participants queued at the end of the class to personally thank the lecturers for their lectures! Videos of these lectures, as well as lectures for the *High dimensional cohomology of moduli spaces* masterclass and workshop on *Homotopy and homology of configuration spaces*, are now available on our GeoTop YouTube channel. Our YouTube channel currently counts > 42.000 views.

1.5 Recruitment and gender strategy

The centre had in 2021, together with the Department of Mathematics, two calls for PhD fellowships and one call for postdoc positions. We took advantage of the international platform MathJobs for increased visibility for the postdoc positions. The postdoc call had more than 400 applicants in pure mathematics, including similarly to last year more than 100 applicants in topology and close to 50 in geometry. **NM Møller** played a coordinating role for the postdoc call for the whole department. For PhD fellowships we had calls for applications in April and in November. The centre had around 30 PhD applicants in each of the spring and the fall application calls this year.

Diversity is on our mind whenever we hire. This applies to both gender diversity and more general diversity in the sense of not always going for the same type of “usual profiles”. We pay much attention to this, including when we choose who to invite for interviews. **Wahl** is part of the department’s diversity committee, where we postponed for one more year (due to Corona) a planned Department-wide activity to raise awareness in diversity issues in the hiring of PhD students and postdocs. We are aware that we still have to work on our gender diversity at the centre. This year, 2 out of 6 new postdocs are female and 0 out of 3 PhD hires were female.

1.6 Research integrity

The three central principles of the Danish research integrity code of conduct are *honesty*, *transparency* and *accountability*. We are fully committed to these three principles and actively work to implement them in several ways.

Mathematics has a tradition of being relatively open in the research process, with researchers sharing their results long before they are in final published form. At GeoTop we ask all centre members to put their preprints on the international preprint archive arXiv.org before submitting them to journals for publication; this promotes transparency. Likewise, staff members present preprints posted to the arXiv at bimonthly centre meetings.

We encourage everyone to publish in high standards international journals that have thorough refereeing processes. Potential issues in mathematical publishing are mistakes (these do happen and need fixing when the author becomes aware of them), failing to give proper credit to people one has received ideas from, or worst, actual plagiarism. The ArXiv online platform automatically checks for text overlap with every article submitted to the platform and adds a comment to the paper's page in case of significant overlap.

The University of Copenhagen also runs compulsory classes on “Responsible conduct in research” for PhD students and PhD supervisors.

2 Research plan

We report here on the progress made on our research plan, under each of the three headings *Moduli*, *Geodesics* and *Singularities*, selecting the most illustrative results.

(A) Moduli

Core: Galatius, Randal-Williams, Wahl

Visiting: Esnault, Le Bras

Postdocs: Bianchi, Biswas, Carmeli, Gritschacher, Iwasa, Jansen, Land, Patzt

PhDs: Asgerisson, Aumonier, Bernard, Cordova, Hilman, Liep, van der Meer, Pacheco-Tallaj, Ramzi, Sroka, Subramanian

The focus on this part of the research has again this year been on Goal A.1, the identification of the homotopy type of moduli spaces.

Much progress has been done in studying moduli spaces of manifolds: **Randal-Williams** and Krannich (former PhD student of **Wahl**) studied the second Weiss derivative of $B\text{TOP}$, complementing earlier work with Kupers on diffeomorphisms of even-dimensional discs. **Randal-Williams** and Kupers studied the Torelli Lie algebra, establishing a property conjectured earlier by Hain, namely that it is Koszul in a stable range. The Torelli group and Torelli Lie algebra are objects from two-dimensional topology, but remarkably their work relied on recent work in high-dimensional manifold theory. Also on the rich topic of 2-dimensional manifolds, **Bianchi** studied generalized Hurwitz spaces, leading to a series of four preprints that give, in particular, a new proof the Mumford conjecture about the homology of the moduli space of surfaces.

On the K -theoretical side, **Patz**, with Kupers and Miller, established an improved range for homological stability of $\text{GL}_n(\mathbb{Z})$. **Carmeli**, with Schlank and Yanovski, continued earlier work on ambidexterity, leading to a third preprint in a series about this topic. PhD student **Aumonier** submitted a paper about an h -principle for moduli spaces of non-singular holomorphic sections of vector bundles, while **van der Meer** and Wong submitted a paper about endotrivial modules via Galois descent.

Three PhD students associated with the centre defended their thesis: **Jansen**, now postdoc at the centre, who wrote a thesis about reductive Borel–Serre compactifications with applications to algebraic K -theory, **Sroka**, whose thesis is concerned with many new patterns in homology of moduli spaces, and **Bernard**, who defended her Stanford PhD thesis studying operations on homology of E_k algebras in symmetric monoidal categories.

Land published a textbook about ∞ -categories, partially based on a PhD course taught in Copenhagen.

(B) Geodesics

Core: Wahl

Postdocs: Avvakumov, Bianchi, Das, Lahtinen, Naef, Razlighi, Schnider, Woike, Zheng

PhDs: Elis, Poliakova, Steinmeyer, Aamand

The focus this year has again been on Goal B.1, concerning geometric string topology, though with some advances on Goal B.2 through part (C), see below.

Ongoing progress on Goal B.1 concerned both the construction of operations (ongoing work of Das and Wahl) and the determination of the geometric meaning of operations, including a preprint released by Naef computing that the string coproduct is not homotopy invariant using lens spaces, a preprint that attracted much attention. In connection with this and related results, Naef, Rivera and Wahl were invited to contribute a survey article on string topology to a volume dedicated to the 80th birthday of Dennis Sullivan, a founder of the field of string topology (and 2022 Abel Prize winner).

Avvakumov, in joint work with Balitskiy, Hubard and Karasev, gave a combinatorial analogue of the classical Riemannian systolic inequality of Gromov relating the length of a shortest geodesic to the volume of a given Riemannian manifold. The combinatorial version has the form of a lower bound on the number of vertices of a simplicial complex in terms of its edge-path systole, i.e. the length of a shortest non-trivial loop consisting of edges. In joint work with Kasarev, Avvakumov also computed bounds on the number of top-dimensional simplices which any triangulation of a space must necessarily have.

Aamand defended in September her master's thesis as part of the 4+4 PhD program, where she shows that solutions to the quantum Yang-Baxter equation can be obtained from the expectation values of crossing Wilson lines in Chern-Simons theory at all orders in perturbation theory. The next step is to consider more complex string interactions. A new PhD student, Elis, was hired to work on describing the string operations on based loop spaces. Poliakova defended her thesis where she studies homotopy monoidal structures associated to derived algebraic groups. The homotopies are described by polytopes that have earlier appeared in Saneblidze's study of free loop spaces.

(C) Singularities

Core: Colding, NM Møller

Postdocs: Friedrich, Lubbe, Ma

PhDs: Muhammad, Zhang

We have this year made progress on both our goals from this part of the proposal.

On Goal C.1, classifying singularities in geometric flows, new progress was made on a question dating at least back to the 1980s concerning the existence and uniqueness of embedded self-shrinking singularities with a rotational symmetry, such as torus singularities in 3-space. Ma, Muhammad and NM Møller in a new preprint show that any such self-shrinking torus has entropy bounded above by an explicit constant. This implies that the space of all such examples is compact, with at most finitely many examples when a further reflection symmetry holds. The proof uses a description of those singularities in terms of geodesics in an associated incomplete Riemannian manifold, tying into Goal B.2 about finiteness properties of geodesics and minimal surfaces. A further consequence is the following: in any (possibly incomplete) surface, with curvature larger than the usual round unit sphere, any closed embedded geodesic curve can have length at most 2π , just like on the sphere. The *embeddedness* is crucial here, as there are infinitely many examples of longer self-intersecting geodesics. Possible connections to Goal B.1 will be investigated, as the string coproduct is exactly looking for essential self-intersections in curves.

There has also been considerable progress on Goal C.2, on the stability and dynamical properties of flows. For cylindrical singularities, PhD student Zhang showed in a new preprint that for a certain

large manifold of initial configuration hypersurfaces, one has asymptotic stability of the rescaled mean curvature flow around the cylinder singularity. This is based on **Zhang**'s more general new results on adiabatic approximation for evolutions, in a separate preprint of his. This robust framework turned out to also be applicable to the area-constrained Willmore flow, in yet another preprint, and to the motion of Ginzburg-Landau vortex filaments. This latter paper is also an important step to answering a well-known question in the mathematical physics of superfluidity in Bose-Einstein condensates, and hence **Zhang** has had this paper accepted by a top mathematical physics journal.

Colding with Minicozzi continued their study of growth of eigenfunctions of drift Laplacians via L^2 -techniques, and applied it to the gauge problem in Ricci flow, i.e. the study of the action of the diffeomorphism group of the underlying manifold on the equations. As an application, they showed that if one tangent flow at a future singular point is a cylinder, then so are all other tangent flows.

3 Comments to the appendix

Appendix—B: Conferences. Despite further challenges caused by the pandemic in 2021, the centre organised 5 events in Copenhagen, with only one fully online, and participated in the organisation of 4 events away from Copenhagen. Note in particular the Oberwolfach seminar on Cellular E_k -algebras, a 1-week event at the German research institute Oberwolfach, dedicated to teaching the recent work of **Galatius**, Kupers and **Randal-Williams** on this subject. See Section 1.4 above for more details about conferences and other activities at the centre.

Appendix—C: Educational activities. The table lists 2 PhD courses, 13 graduate courses and 6 bachelor courses. We note in particular the new version of the “Topics in Geometry” course, taught this year by **NM Møller** together with postdoc **Bianchi**, on interactions between Geometry and Topology, fully in the spirit of the GeoTop centre. The course was very popular, attended by both MSc & PhD students, postdocs and even some permanent faculty.

The summer research experience for undergraduates is under planning, in collaboration with DNRFCentre DAWN, and will have a first run in the Summer 2022.

Appendix—D: External funding. The DNRFC grant is, on top of the support from the Department of Mathematical Sciences, strengthened by 2 grants from the Independent Research Fund Denmark, 2 European Research Council Consolidator grants, and 1 European Research Council Starting grant. Since the start of the centre, we have hosted 5 Marie Skłodowska-Curie stipends. The supplementary external funding in 2021 amounted to 11,5 M DKK.

Appendix—E: Awards. The centre received three invitations to the International Congress of Mathematicians 2022: **Colding** as plenary speaker, **Randal-Williams** and **Wahl** as sectional speakers in the Topology section. This congress takes place every 4 years, and is the most prestigious conference in Mathematics.

Appendix—F: Public outreach. GeoTop made its first appearance at the Culture night, with a mathematical puzzle with a bottle opener, fastest curves in a gravitational field and soap bubbles forming cubes; there were 1838 visitors at the local event. The program “Geometry and Algorithms” of Videnskabsklubben, designed by **NM Møller** and AU computer scientist KG Larsen, where for the design of some practical aspects Copenhagen Game Lab was hired to help, had its first run, with 45 high school students helping 120 or so 6th-graders solving mathematical problems. The centre contributed also with podcasts, an interview of **Clausen** in Yakerson's *Math-Life Balance* series, and consultancy work.

Appendix—H: Publications. We added 37 new preprints to the online preprint database arXiv in 2021 (see <https://geotop.math.ku.dk/research/preprint-arxiv/>) and 9 papers were published in 2021; as publication is generally a very slow process in Mathematics, we do not expect the number of yearly published papers to immediately follow the number of new preprints. The first listed paper appeared

in Journal of the AMS, which is a top-3 journal in Mathematics internationally. The book contribution *Introduction to infinity categories* by Land is partially based on a PhD course given at the centre in 2020.

4 Signature

I hereby confirm the correctness of the information concerning annual accounts, including itemisations. Also, I confirm that the compiled annual reporting, including the appendices, is correct, i.e. it is free of material misstatement or omissions, and that the administration of the funds has been secure and sound, and in accordance with the conditions of the centre agreement.

Date
31/3 2022

Nathalie Wahl



Professor, centre Director